Exercice 1

1) $-8-16+4\times(-2)$

$= -8-16-8$

$= -32$

2) $\frac{-5}{7}\times\frac{2}{3} = -\frac{10}{21}$

$\left(1-\frac{1}{5}\right)\times 6-1$

$= \left(\frac{5}{5}-\frac{1}{5}\right)\times 6-1$

$= \frac{4}{5}\times 6-1$

$= \frac{24}{5}-1$

$= \frac{24}{5}-\frac{5}{5}$

$= \frac{19}{5}$

3) $\frac{15\times 10^{-2}\times 10^{5}}{10^{4}}$

$= 15\times\frac{10^{3}}{10^{4}}$

$= 15\times 10^{-1}$

$= \frac{15}{10}$

$= \frac{5\times 3}{5\times 2}$

$= \frac{3}{2}$

4) $\frac{3x}{4}-\frac{5x}{8}+\frac{2x}{16}$

$= \frac{4\times 3x}{4\times 4}-\frac{2\times 5x}{8\times 2}+\frac{2x}{16}$

$= \frac{12x-10x+2x}{16}$

$= \frac{4x}{16}$

$= \frac{x}{4}$

5) $\frac{1}{4}$ correspond à 25%.

Exercice 2

Partie A :

a) $[-5;2[\cap]-1;4] =]-1;2[$

b) $]-\infty;4[\cap [-2;4] = [-2;4]$

c) $[-5;1] \cap [1;+\infty[= \{1\}$.

Partie B

a) $]-3;2] \cup]-2;5[=]-3;5[$

b) $]-\infty;2] \cup [2;6] =]-\infty;6]$

c) $]-5;1,99] \cup [2;4[=$

Exercice 3

a) $\frac{24,6}{198} = \frac{246}{108} = \frac{2 \times 123}{2 \times 54} = \frac{123}{54} = \frac{3 \times 41}{3 \times 18} = \frac{41}{18} \in \mathbb{Q}$

b) $\frac{\sqrt{56}}{2\sqrt{40}} = \frac{\sqrt{4 \times 14}}{2\sqrt{4 \times 10}} = \frac{2\sqrt{14}}{4\sqrt{10}} = \frac{\sqrt{14}}{2\sqrt{10}} = \frac{\sqrt{2 \times 7}}{2\sqrt{2 \times 5}} = \frac{\sqrt{2} \times \sqrt{7}}{2\sqrt{2} \times \sqrt{5}}$

$= \frac{\sqrt{7}}{2\sqrt{5}} \in \mathbb{R}$.

c) $\frac{15}{25} - \frac{2}{1\sqrt{5}} = \frac{5 \times 3}{5 \times 5} - \frac{2}{1\sqrt{5}}$

$= \frac{3}{5} - \frac{2}{1\sqrt{5}} = \frac{9}{1\sqrt{5}} - \frac{2}{1\sqrt{5}} = \frac{7}{\sqrt{5}} \in \mathbb{Q}$

b) $\frac{21-7\pi}{33-11\pi} = \frac{7(3-\pi)}{11(3-\pi)} = \frac{7}{11} \in \mathbb{Q}$

c) $3(3\sqrt{3}-\sqrt{48})^2 = 3 \times \left((3\sqrt{3})^2 + (\sqrt{48})^2 - 2\times 3\sqrt{3}\times\sqrt{48}\right)$

$= 3\left(9\times 3 + 48 - 6\sqrt{3}\sqrt{3\times 16}\right)$

$= 3(75 - 6\times\sqrt{3}\times\sqrt{3}\times 4)$

$= 3(75 - 6\times 3\times 4)$

$= 3\times 3$

$= 9 \in \mathbb{N}.$

Exercice 4

1) $\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2} = \frac{(n+1)(n+2) - 2n(n+2) + n(n+1)}{n(n+1)(n+2)}$

$= \frac{n^2 + 2n + n + 2 - 2n^2 - 4n + n^2 + n}{n(n+1)(n+2)}$

$= \frac{2}{n(n+1)(n+2)}$

2) $\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2} = \frac{2}{n(n+1)(n+2)}$

donc $\frac{1}{2}\left(\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}\right) = \frac{1}{n(n+1)(n+2)}$

$$S = \frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} + \dots + \frac{1}{30\times31\times32}$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right) + \dots + \frac{1}{2}\left(\frac{1}{30} - \frac{2}{31} + \frac{1}{32}\right)$$

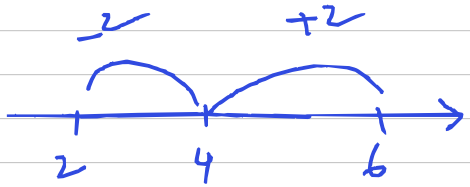
$$= \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4} + \frac{1}{3} - \frac{2}{4} + \frac{1}{5} + \frac{1}{4} - \frac{2}{5} + \frac{1}{6} + \dots + \frac{1}{30} - \frac{2}{31} + \frac{1}{32}\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{3} + \frac{1}{4} - \frac{2}{4} + \frac{1}{4} + \dots - \frac{2}{31} + \frac{1}{32}\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{3} - \frac{2}{31} + \frac{1}{32}\right)$$

Exercice 5

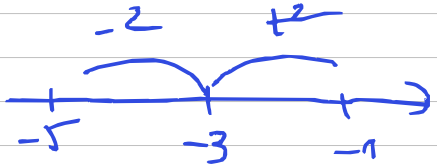
a) $|x-4| = 2$



$S = \{2; 6\}$

b) $|x+3| = 2$

$|x-(-3)| = 2$



$S = \{-5; -1\}$

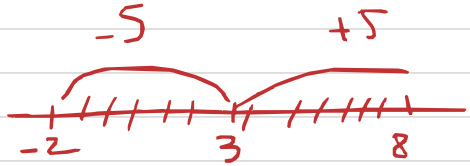
c) $|3x-9| \leq 15$

$|3(x-3)| \leq 3\times5$

$|3|\,|x-3| \leq 3\times5$

$3\,|x-3| \leq 5\times3$

$|x-3| \leq 5$



$S = [-2 ; 8]$

Exercice 7

1) $\vec{KL} = \vec{KG} + \vec{GL}$

$= -\vec{GK} + \vec{GL}$

$= -6\vec{GI} + \vec{GH} + \vec{HL}$

$= -6\vec{GI} + \vec{GH} - 4\vec{HG}$

$= -6\vec{GI} + \vec{GH} + 4\vec{GH}$

$= -6\vec{GI} + 5\vec{GH}$

2) $\vec{JK} = \vec{JI} + \vec{IK}$

$= -\vec{IJ} + \vec{IK}$

$= -\vec{HI} + \vec{IG} + \vec{GK}$

$= -\vec{HI} - \vec{GI} + 6\vec{GI}$

$= -\vec{HI} + 5\vec{GI}$

$= -\vec{HG} - \vec{GI} + 5\vec{GI}$

$= \vec{GH} + 4\vec{GI}$

3) $\vec{KL} = 5\vec{GH} - 6\vec{GI}$

$\vec{JK} = \vec{GH} + 4\vec{GI}$

$5\vec{JK} = 5\vec{GH} + 20\vec{GI}$

$\vec{KL}$ et $\vec{JK}$ ne peuvent pas être colinéaires car on ne peut pas trouver un réel k tel que $\vec{KL} = k\vec{JK}$.

Exercice 8

1) $2x + 3y = 3$ $\qquad xy = -4$

$\Leftrightarrow (2x + 3y)^2 = 3^2$

$\Leftrightarrow 4x^2 + 9y^2 + 12xy = 9$

$\Leftrightarrow 4x^2 + 9y^2 + 12 \times (-4) = 9$

$\Leftrightarrow 4x^2 + 9y - 48 = 9$

$\Leftrightarrow 4x^2 + 9y = 57$

2) $x - y = 4$ $\qquad xy = 4$

$(x-y)^2 = 4^2$

$x^2 + y^2 - 2xy = 16$

$x^2 + y^2 - 2 \times 4 = 16$ $\qquad x^2 + y^2 = 24$